THE REMAINDER R_n for $\sum_{n=1}^{\infty} a_n$ from a Convergent Integral Test Application:

Suppose that sequence $\{a_n\}$, function f, AND Convergent series $\sum_{n=1}^{\infty} a_n = s$ are given as in the statement of the Integral Test.

In the sequence of partial sums, $\{s_n\}$ with $s_n = \sum_{k=1}^{n} a_k$, each s_n can be considered as an approximation of the sum s

THE REMAINDER $R_n = s - s_n = \sum_{k=n+1}^{\infty} a_k$

is the ERROR OF THE APPROXIMATION s_n.

If the Integral Test Applies, with $a_n = f(n)$, THEN

Box(2) on p. 718

$$\int_{n+1}^{\infty} f(x)\,dx \le \underbrace{R_n}_{= s - s_n} \le \int_{n}^{\infty} f(x)\,dx = \text{MAXIMUM POSSIBLE ERROR in } s_n$$

AND

BOX (3) on p. 719

$$\underbrace{\left(s_n + \int_{n+1}^{\infty} f(x)\,dx\right)}_{A} \le s \le \underbrace{\left(s_n + \int_{n}^{\infty} f(x)\,dx\right)}_{B}$$

Midpoint $m = \frac{A+B}{2}$ is the improved approximation, $m \approx s$.

REMAINDER R_n BOUNDS

$$R_n = \sum_{k=n+1}^{\infty} a_k = s - s_n$$

$$R_n \leq \int_n^{\infty} f(x)\,dx$$

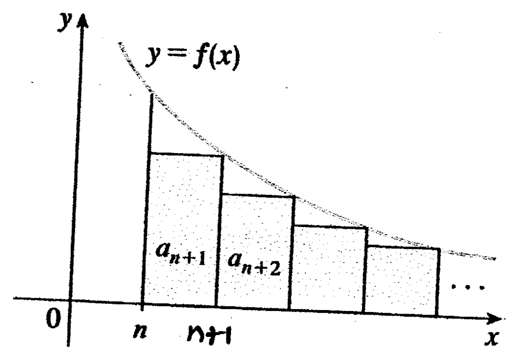

FIGURE 3

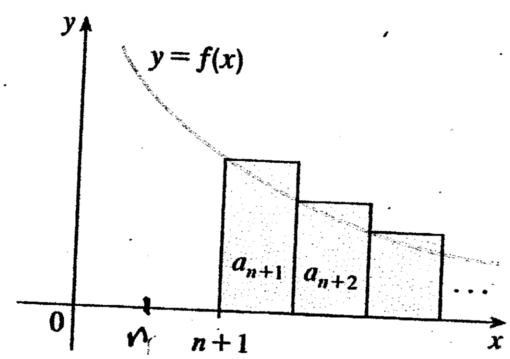

FIGURE 4

$$\int_{n+1}^{\infty} f(x)\,dx \leq R_n$$

So, $\int_{n+1}^{\infty} f(x)\,dx \leq R_n \leq \int_n^{\infty} f(x)\,dx$